Transport anomalies of the strange metal: Resolution by hidden Fermi liquid theory

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The strange metal phase of optimally and overdoped cuprates exhibits a number of anomalous transport properties—unsaturating linear-T resistivity, distinct relaxation times for Hall angle and resistivity, temperature-dependent anisotropic relaxation times, and a characteristic crossover from supposed Fermi liquid to linear-T behavior. All receive natural explanations and quantitative fits in terms of the hidden Fermi liquid theory.

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I. INTRODUCTION

From the very first observations of the properties of the cuprate "high-T_c" superconductors it was seen that the properties of the "normal" metal above T_c were unusual. There are actually two unusual regimes: at lower doping, there develops a "pseudogap" regime which is most plausibly described[1,2] as a state with BCS pairing but without superconducting order. (At still lower dopings various complex phases with inhomogeneities and/or alternative orderings show up also, but we will consider only homogeneous translationally symmetric phases.) Near optimal and above (and also for T above the pseudogap regime) there is no evidence of pairing in the normal state but instead a characteristic "strange" metallic behavior extending to very high temperatures and energies. Most obvious is the notorious "linear-T" resistivity, sometimes extrapolating at $T=0$ to zero or less and persisting in its linearity often to well above the Mott limit. A cleaner characterization, if available, is the "Drude-like" tail of the midinfrared conductivity. This falls off as a noninteger power of frequency considerably less than the ω^{-2} of the Drude theory.[3,4]

Very early a heuristic for the strange behavior was developed as the "marginal Fermi liquid" theory[5] and that is often used as a descriptive term, but this heuristic does not describe the infrared result correctly nor any of the further irregularities. Another heuristic which has been proposed to describe some of the anomalies is the idea of "hot spots" and "cold spots" on the Fermi surface at which different temperature dependences are observed.[6–8] Aside from difficulties in fitting the transport phenomena this makes no reference to the anomalous power laws in the infrared which we use as the defining characteristic of the strange metal and has no microscopic explanation which we find convincing of the non-Fermi liquid behavior.

Often a third regime is postulated, which the state returns to the simple Fermi liquid when overdoped beyond the superconducting dome, and for lower T, a crossover line being drawn up and to the right, starting at the edge of the dome. We will see that this is merely a crossover in the transport properties and those fundamental properties such as the one-particle Green's function remain anomalous according to our theory. No transition to a true Fermi liquid has been observed in our opinion.

A striking anomaly of the strange phase is the T-dependent Hall effect. It is best described as there existing a relaxation rate for the Larmor precession $1/\tau_H$ distinct from that for the resistivity and more resembling that for a Fermi liquid.[9,10]

The purpose of the rest of this paper is to show how all of these anomalies follow from the theory of the simplest possible model, the Hubbard model with a strong interaction U and *nothing else*.

II. HIDDEN FERMI LIQUID

The hidden Fermi liquid (HFL, hereafter) theory[11] depends on the assumption that the Hubbard on-site interaction U is sufficiently strong that it must be renormalized to infinity by the Gros-Rice canonical transformation, leaving behind a superexchange interaction and the kinetic energy projected on the lower Hubbard band. That is, the effective Hamiltonian is

$$H = \sum_{i,j} J_{ij} S_i \cdot S_j + P\left[\sum_{i,j,\sigma} t_{ij} c^*_{i,\sigma} c_{j,\sigma}\right] P,$$

$$P = \prod_i (1 - n_{i\uparrow} n_{i\downarrow}). \tag{1}$$

This "t-J Hamiltonian" is not simply a convenient alternative to the Hubbard model; it reflects the physical fact that the low-energy states live within a subspace which is overcompletely described by a single full band of electron states because antibound states (doublons) have been ejected out of the top of the band. No convergent perturbative route exists to connect the low states to the original band of the Hubbard model since they exist within Hilbert spaces of different dimensionalities.

It is assumed that in the strange metal region J is too weak because of competition with kinetic energy[12] or thermal fluctuations to cause pair condensation and an anomalous self-energy, and therefore its major effect can be lumped in with that of phonons as a renormalization of the kinetic energy. It will also contribute electron-electron scattering but we do not expect it to be as large as that due to the projection. Therefore the problem reduces to the effect of Gutzwiller projection on the renormalized kinetic energy, represented by a simple Fermi gas, that is, to the second term in H, so we consider the Hamiltonian

$$H_p = P\sum_{i,j,\sigma} t_{ij} c^*_{i\sigma} c_{j\sigma} P = \sum_{i,j,\sigma} t_{ij} \hat{c}^*_{i,\sigma} \hat{c}_{j,\sigma},$$

$$\hat{c}_{i\sigma} = c_{i\sigma}(1 - n_{i,-\sigma}), \tag{2}$$

where we introduce the projective quasiparticle operators $\hat{c}$ and $\hat{c}^*$, which automatically enforce the projection.

The HFL ansatz is that projected Hamiltonian (2) operating in the *unprojected* Hilbert space of many-electron wave functions gives one the low-energy spectrum of a Fermi liquid essentially, which it has a sharp Fermi surface with the usual analyticity properties of the self-energies of the quasiparticles c and c^*. The ansatz can be thought of as the result of a Shankar-style[13] renormalization but can really be justified only by demonstrating its self-consistency and by testing to what extent it agrees with experiment; in both respects it seems so far to have passed muster. But the quasiparticles in this Hilbert space are not the true quasiparticles of the physical system: these are the projected quasiparticles which we designate with "hats." We shall hereafter invent the name "pseudoparticle" to describe the objects c and c^* which obey Fermi liquid rules because they operate in the full Hilbert space. The pseudoparticles have renormalized Fermi velocities which can be estimated with the Gutzwiller approximation,

$$v_{F,ren} = v_{F,0} g_t,$$

$$g_t = 2x/(1+x), \tag{3}$$

with x being the doping percentage. They can be expected to have rather large electron-electron scattering, Γ_{ee}, proportional to $(k-k_F)^2$. In the one case in which we have accurate information, optimally doped $Bi_2Sr_2CaCu_2O_{8+\delta}$ (BiSCCO),[14] the coefficient is

$$\Gamma_{ee} = C v^2_{F,ren} (k - k_F)^2,$$

$$C = 3.6 \times 10^{-3}/\mathrm{meV}. \tag{4}$$

Straightforward phase space considerations would, as observed by Zheleznyak *et al.*,[15] suggest that the coefficient should be of the order $1/W$, with W being the bandwidth, but in the Hall effect case of interest to him he observed that it was considerably larger, and we also find this: W is of order a few hundred kelvins rather than a few thousand. A little thought persuades us that this should be the case. The Gutzwiller projection slows the coherent Fermi velocity for an electron with spin near the Fermi surface, but it does not much affect the incoherent motions of bare holes, which are just as rapid as in the unprojected state—for instance, the second moment of the overall spectrum is unaffected.[16] The quasiparticles are broadened by these incoherent motions proportionately to this second moment, roughly, so one might expect that the broadening would be proportional to g^{-2} or about one order of magnitude larger than the naive estimate. Thus our hidden Fermi liquid will tend not to be a very good one in the sense that the coherence of its pseudoparticles lasts only out to about 50 meV from the Fermi surface. We should also note that there is no reason to expect this scattering mechanism to be anisotropic.

Let us now consider the transport properties of such a system: first the resistivity. As Anderson discussed in his book and in related papers,[17,18] this is complicated by being a two-step process. The momentum is delivered to the system via accelerating the true quasiparticles, i.e., by displacing their Fermi surface. But the scattering which transfers momentum to the lattice is the T^2 umklapp scattering of the pseudoparticles which we have just been discussing. Gutzwiller projection is perfectly translation invariant, so that the process of decay of true quasiparticles into pseudoparticles is momentum conserving and cannot lead to resisivity by itself. It acts, instead, as a bottleneck, a necessary step which must take place before the true scattering events can operate. (As Anderson noted in Ref. 12, this is actually the same physics which is involved in phonon drag, but we think the "bottleneck" description is clearer.) It is the slower of the two processes which will control the rate: they do not add according to Matthiessen's rule but according to its inverse.

In previous work[12] Anderson calculated the dissipation due to the quasiparticle decay process by approximating the two-particle Green's function which appears in the response function by the simple product of two one-particle functions since it should be a good approximation for the quasiparticles to decay independently. In Ref. 11 and related papers[14] we have shown that the form of the single-particle Green's function at absolute zero is the simple expression,

$$G(r,t) = G_0(r,t) G^*(t) \begin{Bmatrix} 1 \\ g \end{Bmatrix},$$

$$G^*(t) = t^{-p}, \quad \text{where } p = (1-x)^2/4. \tag{5}$$

Here the 1 applies on the hole side and the g on the electron. (For finite T, presumably, the jump singularity of the coefficient becomes a Fermi function.) G_0 is the pseudoparticle Green's function. In Ref. 11 we showed how to generalize Eq. (5) to finite temperature. G_0 follows the conventional rules, while as we pointed out there, the power law in G^* was shown by Yuval and Anderson[19] to follow the general rule of being antiperiodic in imaginary time by becoming

$$G^*(r,t) = \left(\frac{\pi T}{\sinh(\pi T t)}\right)^p \cong e^{-\pi p T t} \quad \text{for } Tt \gg 1. \tag{6}$$

This is the source of the ubiquitous linear-T decay. Note that the relaxation rate is isotropic, but the mean free path and therefore the conductivity will have the anisotropy of the Fermi velocity since the Fermi *momentum* is fairly isotropic.

At high frequencies and high temperatures the T^2, ω^2 decay implied by Eq. (4) may be assumed to be more rapid than Eq. (6) and dissipative processes will be dominated by the power law decay of quasiparticles into pseudoparticles. The most straightforward situation is the infrared conductivity which has long been known to obey a frequency power law,[20]

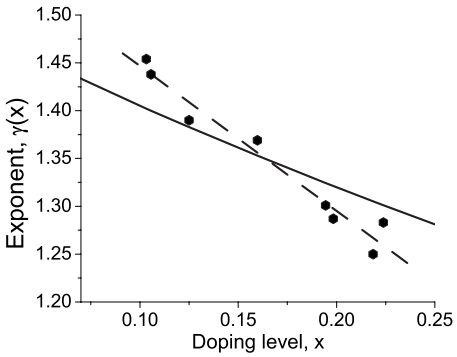

FIG. 1. Infrared spectrum exponents for $Bi_2Sr_2CaCu_2O_{8+\delta}$. Data points from Ref. 21 with linear best fit in Ref. 21 (dashed line) and predicted value from Ref. 30 (solid line). The predicted exponent stems from $\sigma(\omega)=(i\omega)^{-2+\gamma}$ with $\gamma=1+2p$ and p is given in Eq. (5). Figure reproduced from Ref. 14.

$$\sigma_{ir}(\omega) \propto (i\omega)^{-1+2p}, \tag{7}$$

which can easily be derived from Eq. (5).

Hwang *et al.*[21] has experimentally estimated the dependence of the power $2p$ on doping, which we show in Fig. 1; the agreement as to magnitude is good and the dependence on doping is a bit slow. But our prediction is within the scatter of the data.

As far as dc resistivity is concerned, Eq. (6) accounts for the observed linear dependence on T near-optimal doping. The trend with doping is in agreement with the expected $(1-x)^2$ dependence of p, though in order to be quantitative one would need an estimate of the carrier density which is hard to come by.

In the same regime we see the striking phenomenon first observed by Chien *et al.* (see Ref. 9) of a qualitative difference between the relaxation times τ as estimated from the dc conductivity using $\sigma=ne^2\tau/m$, as opposed to using the Hall angle formula $\Theta_H=\omega_c\tau_H$. The latter shows a conventional Fermi liquid temperature dependence $\propto T^2$, while the resistivity is linear in T as we have just been describing. In the HFL theory this difference is very natural: the Hall angle observed is that of the underlying pseudoparticles of the HFL. The Larmor precession which is caused by the magnetic field does not change relative occupancies and therefore does not disturb the equilibrium between quasiparticles and pseudoparticles: effectively, the magnetic field commutes with Gutzwiller projection. Thus the Hall effect and other magnetic responses—such as the de Haas–van Alfven effect—will be identically those of the HFL, with no bottleneck caused by the decay of the quasiparticles.

The only effect of the strong interaction will be quantitative. As we remarked above, the T^2 relaxation rate will be unexpectedly large. We have compared the temperature dependence of the pseudoparticle lifetime from angle-resolved photoemission spectroscopy (ARPES) in optimally doped BiSCCO and from the dc Hall effect of optimally doped $Bi_2Sr_{2-x}La_xCuO_6$ (BiSLCO) in Ref. 22. Starting from the ARPES result of Eq. (4) for the pseudoparticle scattering rate, we found $\tau_{\mathrm{HFL}}^{-1}=T^2/W$ with $W\approx 500$ K. A precise numerical fit would involve a very complete study of the Fermi surface curvature, anisotropy of the Fermi velocity, and cyclotron frequency, but the Hall angle data lead to a bandwidth quite consistent with 500 K.

The final topic to take up is the resistivity in the region completely beyond the "dome" which is normally designated as "the Fermi liquid."[23] Indeed, the resistivity at low temperatures seems to obey the T^2 law; but we see no reason to suppose that the effects of the strong interaction die out so suddenly. Actually, the resistivity in this region seems to be nicely explained in terms of the bottleneck effect along with the anisotropy of the HFL conductivity due to the anisotropy of v_F.

The temperature dependence of the resistivity, then, is obtained by combining the two conductivities,

$$\sigma_{\mathrm{HFL}} = ne^2\tau/m = \frac{e^2}{\hbar}\frac{\hbar^2 k_F^2}{m}\frac{\tau}{\hbar} = \frac{e^2}{\hbar}\frac{E_F W}{T^2}. \tag{8}$$

Here we have ignored numerical factors of order 1, realizing that they may be subsumed in the parameter W, the effective bandwidth discussed under Eq. (4). Conductivities are two dimensions, per single plane, and T is in energy units. The effective conductivity corresponding to the decay process [Eq. (6)] is

$$\sigma_{decay} = \frac{ne^2 v_F \tau}{m v_F} = \frac{e^2(\hbar k_F v_F)}{\hbar T} = \frac{e^2}{\hbar}\frac{E_F}{T}(v_F/v_{F0}). \tag{9}$$

Here v_{F0} is the maximum Fermi velocity, which gives us an estimate of the overall bandwidth E_F; then we make explicit the dependence on Fermi velocity which will indeed vary quite strongly from the diagonal direction to the zone corners (and in the right direction to account for the anisotropy observed by Hussey[24]).

First we would like to compare the general temperature dependence of the resistivity implied by Eqs. (8) and (9) with relatively early measurements on overdoped cuprates, where there was no attempt to disentangle the anisotropy.[23,25,26] In this case, leaving out the anisotropic Fermi velocity, the resistivity is the universal expression,

$$\rho = \frac{\hbar}{e^2 E_F}\frac{T^2}{T+W}(+\rho_{res}),$$

$$d[\ln(\rho-\rho_{res})]/d(\ln T) = 1 + W/(T+W) \tag{10}$$

(some samples show a small residual resistance which we would expect to be simply additive as la Matthiessen's rule, playing no role in the bottleneck.) The fit of the form [Eq. (10)] to the data is quite satisfactory. For instance, in Ref. 26 there is a plot of the effective exponent vs T, which for low T, where the data are most accurate, follows the second equation of Eq. (10) accurately. Manako *et al.*[26] fitted the data over the entire range with a $T^{3/2}$ power law, which according to Eq. (10) should only be approximate; indeed, we get as accurate a fit, except at high T, where the measurement

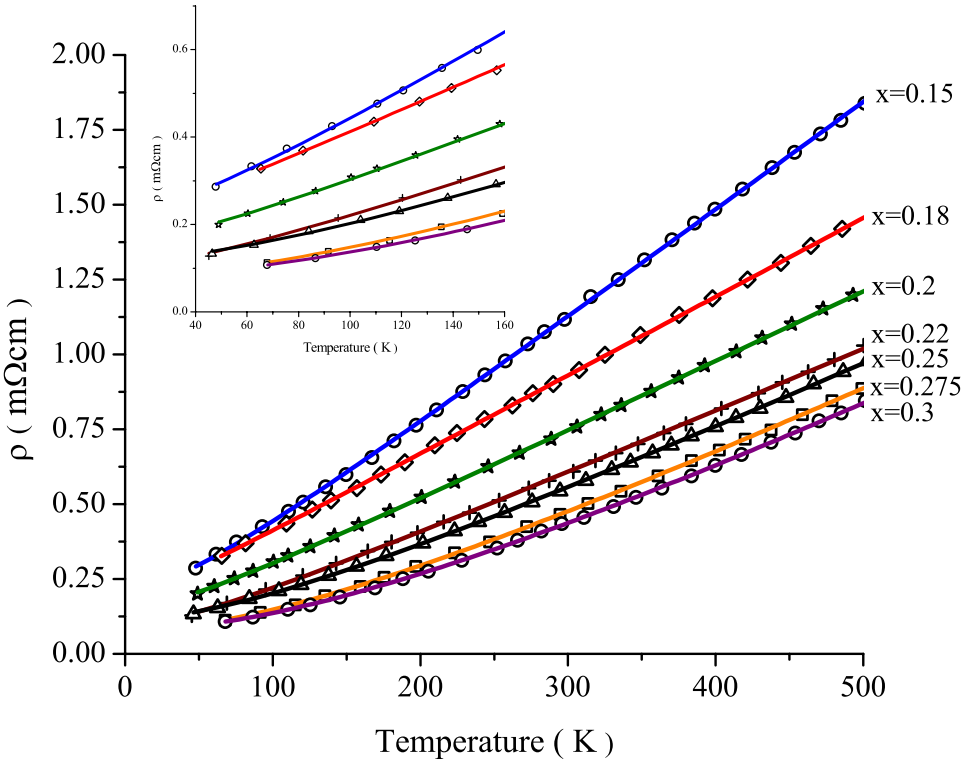

FIG. 2. (Color online) Comparison of the polycrystalline $La_{2-x}Sr_xCuO_4$ resistivity (data points) extracted from Ref. 23 with the bottleneck resistivity form of Eq. (10). Inset slows the low temperature region in detail.

is questionable because of thermal expansion. Figure 2 shows our fit to the data in Ref. 23 and Fig. 3 shows the values of the parameters in Eqs. (8) and (9) obtained from the fit as a function of doping.

The x dependence of the parameter W is experimentally even stronger than x^2. We find the near vanishing of the parameter W for near-optimal doping rather puzzling, especially since it conflicts with our putative conclusion from the Hall effect anomaly, which the T^2 resistance, while high, is finite. Resistivity under a 60 T magnetic field in BiSLCO (Ref. 27) does indicate that W is on the order of 250 K near-optimal doping. This is notably larger than the LSCO results shown in Fig. 3(a) and more consistent with the bandwidth from the BiSLCO Hall and BiSCCO ARPES analysis. But there are a number of assumptions we have made, any of which may affect the prediction for these superconducting samples: we are ignoring the possibility of fluctuation conductivity, and we do not really know precisely how to include residual resistivity.

Abdel-Jawad *et al.*[28] provided an even more explicit confirmation of our theory. Hussey's equation [Eq. (3) in Ref. 24] shows that he is empirically driven to the necessity of adding conductivities [Eqs. (8) and (9)], rather than resistivities, but unfortunately not in quite the correct form [Eq. (10)]. His work using angle-dependent magnetoresistance measurements[28] has shown experimentally that in the optimal-to-overdoped regime, there are two scattering mechanisms for every momentum on the Fermi surface (not "hot" and "cold" spots) with distinct temperature and angle dependences, and as we pointed out above the theory provides precisely those temperature dependences and the cor-

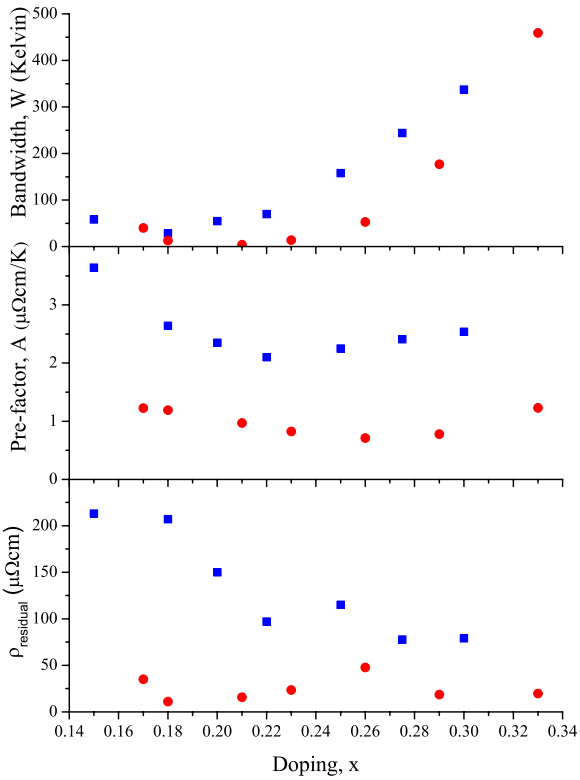

FIG. 3. (Color online) Parameters of the bottleneck resistivity form of Eq. (10), $\rho = AT^2/(T+W)+\rho_{res}$, for comparisons in Fig. 2 (blue squares) and Fig. 4 (red circles). The three parameters are (a) the bandwidth, (b) a prefactor for the first term in Eq. (10), and (c) the residual resistivity.

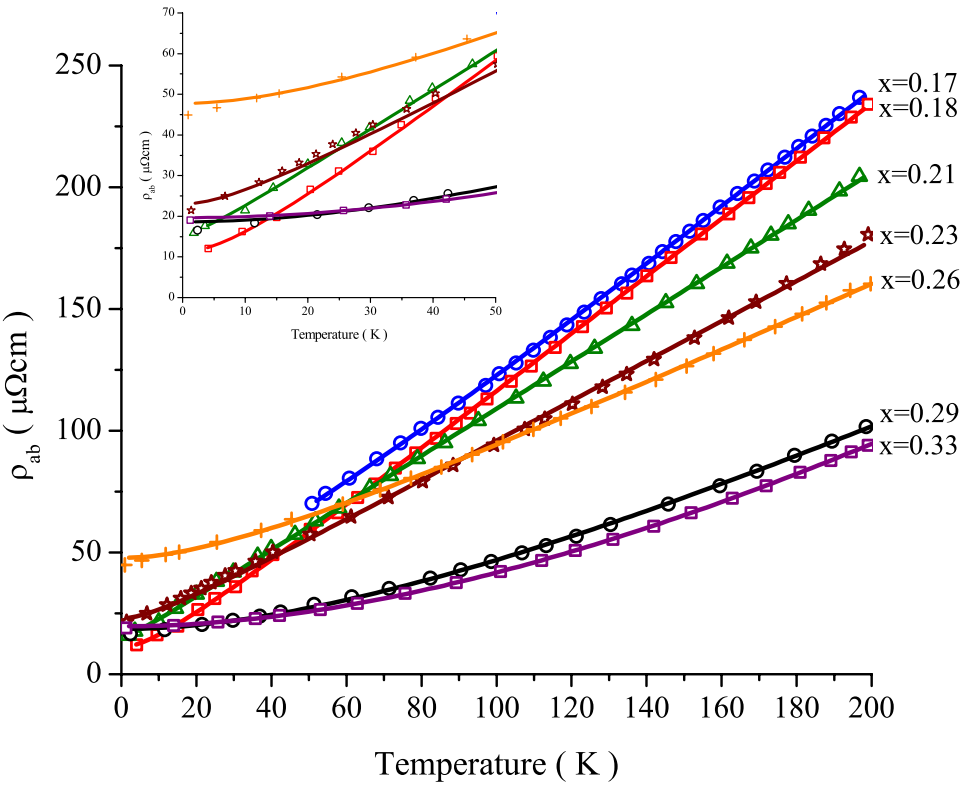

FIG. 4. (Color online) Comparison of the single crystal $La_{2-x}Sr_xCuO_4$ resistivity (data points) extracted from Ref. 29 with the bottleneck resistivity form of Eq. (10). Functional parameters can be found in Fig. 3. Inset shows the low temperature region in detail. The low temperature resistivity data were determined by Hussey *et al.* by suppressing superconductivity with a large magnetic field and then extrapolating the high field resistivity data to zero field.

rect sign and magnitude for the anisotropy of the linear-T term.

In a very recent paper,[29] the same group have revisited the doping range in Ref. 23, focusing on temperatures below 200 K. They have used a large magnetic field to destroy superconductivity when present so have a lower minimum temperature. Their fitting function is purely empirical and has more parameters to adjust than Eq. (10), and in fact we can achieve an equal level of agreement over their low temperature range (see Fig. 4 and parameters in Fig. 3). The magnetic field adds to the uncertainties as to the value of the parameter W in the superconducting range.

III. CONCLUSION

The hidden Fermi liquid method seems well on the way to providing a complete resolution of the anomalous properties of the "strange metal" phase of the cuprate superconductors. Complex seeming as they are, these seem to follow from the slightest possible generalization of the conventional Fermi liquid theory of metals, namely, the inclusion of the projective constraint made necessary by the existence of strong on-site electron-electron interactions. This simple case, far from being an impenetrable mystery as it is so often pictured to be, should provide the canonical model for more complex examples of strongly interacting electronic systems.

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